Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 1	DAILY TUTORIAL SHEET 1

1.(C) - - - - -

Numbers divisible by 4 should have last two digits divisible by 4. *i.e.* it should be 00,04, 12, 20, 24, 32, 40, 44, 52

Total number of five-digit numbers $\underline{5}\ \underline{6}\ \underline{6}\ \underbrace{\underline{1}\ \underline{1}}_{9 \text{ ways}}$ $\Rightarrow$ 5 × 6 × 6 × 9 = 1620

2.(A)

0, 1, 2, 3, 4, 5

$\rightarrow$ 0, 1, 2, 3, 4 × sum should be divisible by 3

0, 1, 2, 3, 5 ×

0, 1, 2, 4, 5 ✓ $\rightarrow$ case I : $\underline{4}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 96$

0, 1, 3, 4, 5 ×

0, 2, 3, 4, 5 ×

1, 2, 3, 4, 5 ✓ $\rightarrow$ case I : $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 120$

$\Rightarrow 216$

combining both cases, total number of numbers = 216

3.(C) Divisible by $6 \longrightarrow$ Divisible by 3 and should be even.

Consider (Question - 2)

0, 1, 2, 4, 5 $\quad \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \frac{1}{0} = 24$

Case I : 0, 1, 2, 4, 5 $\rightarrow$ $\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \frac{1}{0} = 24$

$\rightarrow$ $\underline{3}\ \underline{3}\ \underline{2}\ \underline{1}\ \frac{2}{2,\ 4} = 18 \times 2$

Case II : 1, 2, 3 4, 5 $\quad \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \frac{2}{2,\ 4} = 48$

Total number of numbers = 60 + 48 = 108

4.(D) $\underline{o}\ \underline{e}\ \underline{o}\ \underline{e}\ \underline{o}\ \underline{e}\ \underline{o}$ Total number of numbers = $\frac{4!}{2!2!} \times \frac{3!}{2!} = 18$

$e \equiv$ even, $o \equiv$ odd

5.(B)

0, 2, 4, 6, 8

0, 2, 4, 6 ✓ $\rightarrow$ case I : $\underline{3}\ \underline{3}\ \underline{2}\ \underline{1} = 18$

0, 2, 4, 8 ×

0, 2, 6, 8 ×

0, 4, 6, 8 ✓ $\rightarrow$ case I : $\underline{3}\ \underline{3}\ \underline{2}\ \underline{1} = 18$

2, 4, 6, 8 ×

$\Rightarrow 36$

Hence total number of numbers = 36

6.(C) Atleast one dice with odd number = Total ways – [No dice showing odd] = $6^n - 3^n$

7.(C) $1 \longrightarrow 100$, 20 times 5 occurs

$101 \longrightarrow 200$, 20 times 5 occurs

⋮

$901 \longrightarrow 999$, 20 times 5 occurs

But in 500 series, 5 will occur at hundreds place for 100 times. Hence total number of times 5 repeats = 20 × 10 + 100 = 300

8.(D) Required number of numbers = Total number of numbers – Number with no repeated digit

= $10^5 - 10 \times 9 \times 8 \times 7 \times 6$

$\downarrow$

(zero is allowed on 1st place) = 69760

9.(B) - - - - - -, 3 heads, 3 tails $\equiv \dfrac{6!}{3!3!} = 20$

10.(B) $\underline{3}\ \underline{5}\ \underline{5}\ \underline{5}$ Total number of numbers = $3 \times 5 \times 5 \times 5 = 375$

$\uparrow$ $\boxed{1, 2, 3}$

11.(A) Since ${}^mC_{r-1} + {}^mC_r = {}^{m+1}C_r$ we write the given inequality as ${}^nC_6 < {}^nC_7$

$\Rightarrow \quad \dfrac{{}^nC_6}{{}^nC_7} < 1 \qquad \Rightarrow \quad \dfrac{7}{n-6} < 1 \quad \Rightarrow \quad n > 13 \qquad \therefore \qquad$ The least value of n is 14.

12.(B) We have: $15 = 3 \times 5$

Now, $\quad E_3(100!) = 48 \quad$ and $\quad E_5(100!) = \left[\dfrac{100}{5}\right] + \left[\dfrac{100}{5^2}\right] = 20 + 4 = 24$

$\therefore \qquad$ Exponent of 15 in 100! = min (24, 48) = 24

13.(C) $\{0^2, 1^2, 2^2, 3^2, 4^2\} \equiv \{0, 1, 4, 9, 16\} \quad \Rightarrow \{0, 0\}, \{1, 4\}, \{1, 9\}, \{4, 16\}, \{9, 16\}$ Hence (5) ways

14.(C) $A \equiv \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

When numerator and denominator are diff., number of ways = 10 × 9 = 90

When numerator and denominator are same, $\dfrac{\text{Num.}}{\text{Den.}} = 1$ can be formed in 1 way

Total rational numbers = 90 + 1 = 91

15.(D) $\underline{3}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{2}$

2, 4, 6 $\uparrow$ $\quad$ $\uparrow$ 2, 4, 6 Total number of numbers = 3 × 5 × 4 × 3 × 2 × 2 = 720

16.(D) First select 2 seats for women in ${}^4C_2$ ways and arrange them in 2! ways.

Then select 3 seats for men in ${}^6C_3$ ways and arrange them in 3! ways.

Total ways = $({}^4C_2 \times 2!) \times ({}^6C_3 \times 3!) = {}^4P_2\ {}^6P_3$

17.(B) 1, 2, 3, 4, 5 $\longrightarrow$ atleast 2 digits are identical $\equiv$ Total number of numbers – Numbers with no repeated digit = $5^4 - 5! = 505$

18.(C) **Case I:** alphabet only $\to$ 26 ways $\qquad$ **Case II:** alphabet, then digit $\to$ 26 × 10 ways = 260

Total ways = 260 + 26 = 286 ways.

19.(B) Number of diagonals $= {}^{15}C_2 - 15 = 90$

20.(B) We have ${}^{47}C_4 + \displaystyle\sum_{j=1}^{5} {}^{52-j}C_3$.

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 = {}^{47}C_3 + {}^{47}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= \left({}^{47}C_3 + {}^{47}C_4\right) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 \qquad \left[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r\right]$

$= \left({}^{48}C_3 + {}^{48}C_4\right) + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 \qquad \left[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r\right]$

$= {}^{49}C_4 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= \left({}^{49}C_3 + {}^{49}C_4\right) + {}^{50}C_3 + {}^{51}C_3 \qquad \left[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r\right]$

$= {}^{50}C_4 + {}^{50}C_3 + {}^{51}C_3$

$= \left({}^{50}C_3 + {}^{50}C_4\right) + {}^{51}C_3 \qquad \left[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r\right]$

$= {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$ $\quad\left[\because\ {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r\right]$

21.(B) Here, we use ${}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r$ $\quad$ **. . . .(i)**

Now, ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2\ .\ {}^{n}C_r$

$= {}^{n}C_{r+1} + {}^{n}C_{r-1} + {}^{n}C_r + {}^{n}C_r = {}^{n}C_{r+1} + {}^{n}C_r + {}^{n}C_r = {}^{n+1}C_{r+1} + {}^{n+1}C_r = {}^{n+2}C_{r+1}$

22.(C) ${}^{7-x}P_{x-3}$ is defined when $x-3 \le 7-x \Rightarrow x \le 5$ and $x-3 \ge 0 \Rightarrow x \ge 3$.

$\Rightarrow$ The possible values of ${}^{7-x}P_{x-3}$ are when $x = 3, 4$ or 5.

$\Rightarrow$ The range of values of ${}^{7-x}P_{x-3}$ is $\left\{{}^{4}P_0,\ {}^{3}P_1,\ {}^{2}P_2\right\}$ or {1, 3, 2}.

23.(C) It is obvious and can be checked by putting the values. Since other three sets do not hold good.

24.(D) $\dfrac{{}^{5}C_n}{{}^{4}C_n} = 1 + \dfrac{{}^{5}C_n}{{}^{6}C_n} \Rightarrow \dfrac{5}{5-n} = 1 + \dfrac{6-n}{6} \Rightarrow n^2 - 17n + 30 = 0 \Rightarrow n = 2$

25.(A) ${}^{n}C_5 + {}^{n}C_6 > {}^{n+1}C_5 \Rightarrow {}^{n+1}C_6 > {}^{n+1}C_5$

$\Rightarrow \dfrac{(n+1)!}{6!.(n-5)!} . \dfrac{5!.(n-4)!}{(n+1)!} > 1 \Rightarrow \dfrac{(n-4)}{6} > 1 \Rightarrow n-4 > 6 \Rightarrow n > 10$

Hence according to options $n = 11$